Robotics

Chapter 3
Manipulator Kinematics

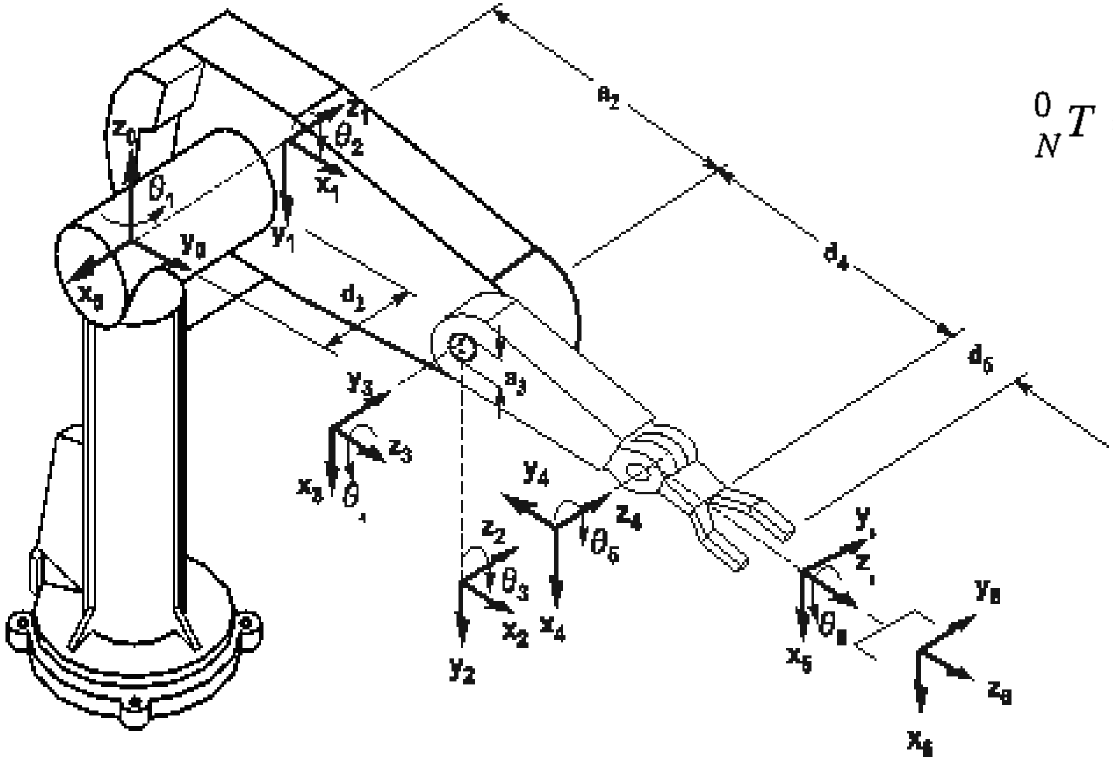
Kinematics

- Kinematics is the study of motion without regard for the forces that cause it.
- It refers to time-based and geometrical properties of motion.
- It ignores concepts such as torque, force, mass, energy, and inertia.

Forward Kinematics

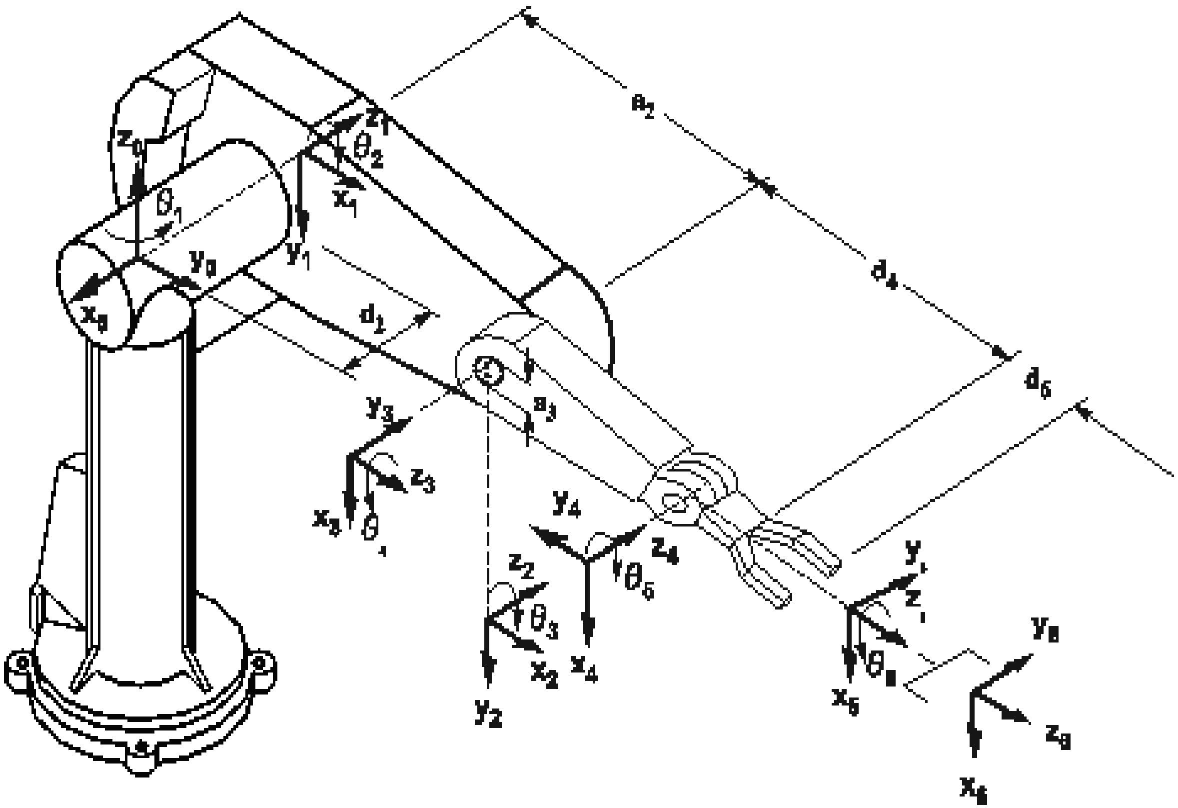
- For a robotic arm, this would mean calculating the position and orientation of the end effector given all the joint variables.

Position and orientation of end effector (x,y,z) w.r.t {base}=f(θ1, θ2, θ3…. Θn)
Note: assuming that all joints are revolute.



$$ {}^{0}_{N}T = {}^{0}_{1}T\, {}^{1}_{2}T\, {}^{2}_{3}T \ldots {}^{N-1}_{N}T. $$

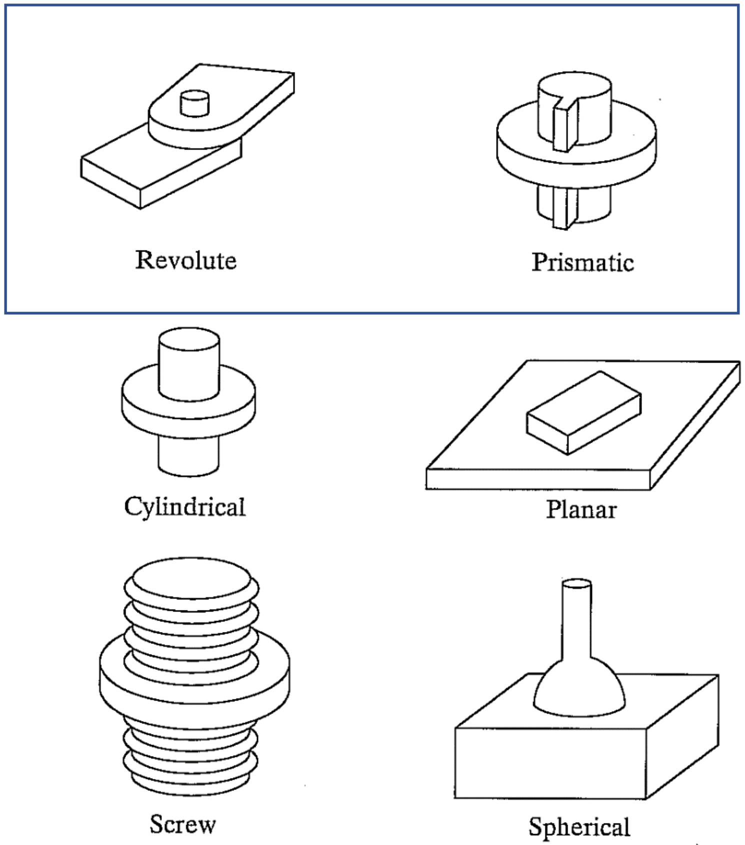
- Inverse Kinematics is the reverse of Forward Kinematics. (!)
- It is the calculation of joint values given the positions, orientations, and geometries of mechanism's parts.
- It is useful for planning how to move a robot in a certain way.

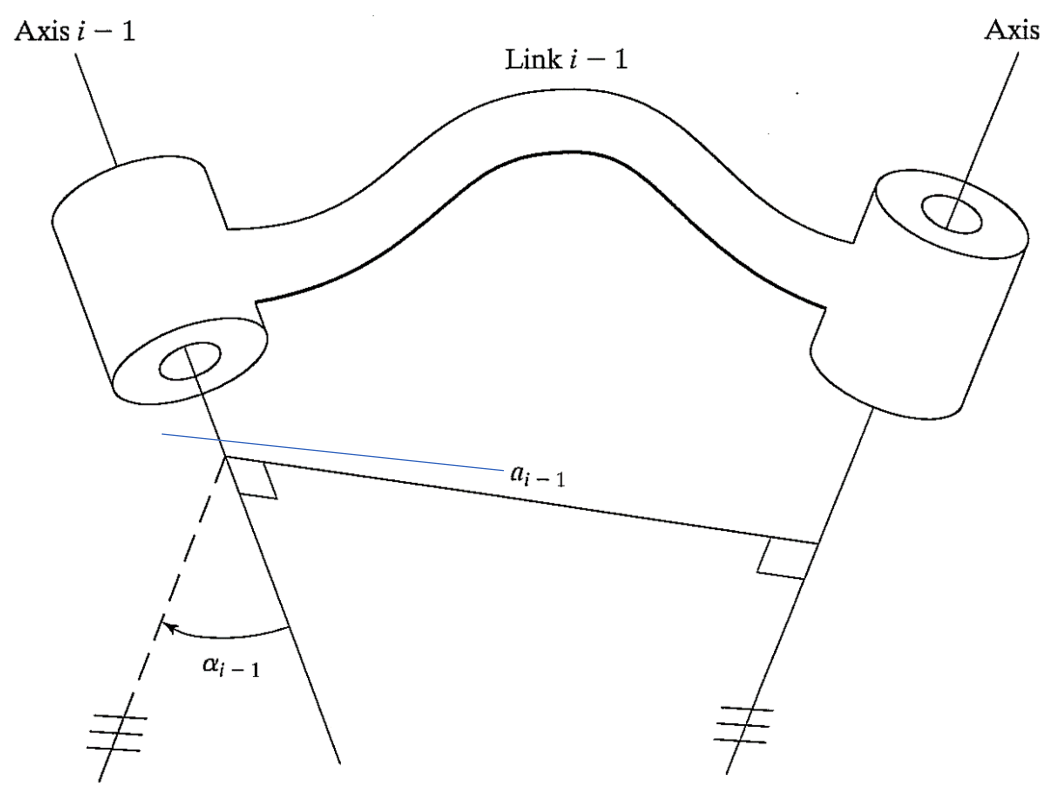


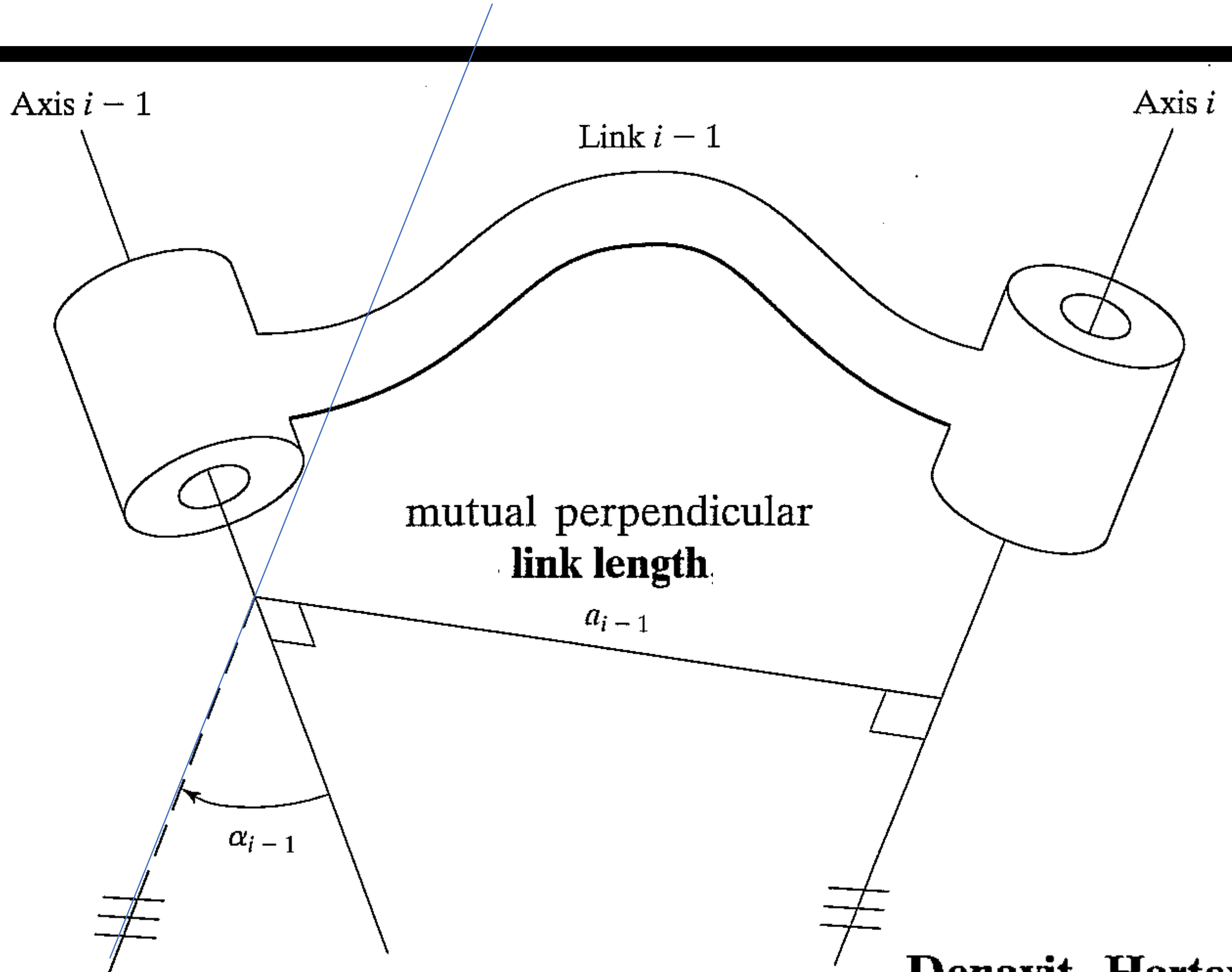
(θ1, θ2, θ3…. Θn) = f(Position and orientation end effector(x,y,z)

3.2 LINK DESCRIPTION

A manipulator may be thought of as a set of bodies connected in a chain by joints. These bodies are called links. Joints form a connection between a neighboring pair of links.







Denavit–Hartenberg notation

link twist.

This angle is measured from axis $i-1$ to axis i in the right-hand sense about a_{i-1}.

3.3 LINK-CONNECTION DESCRIPTION

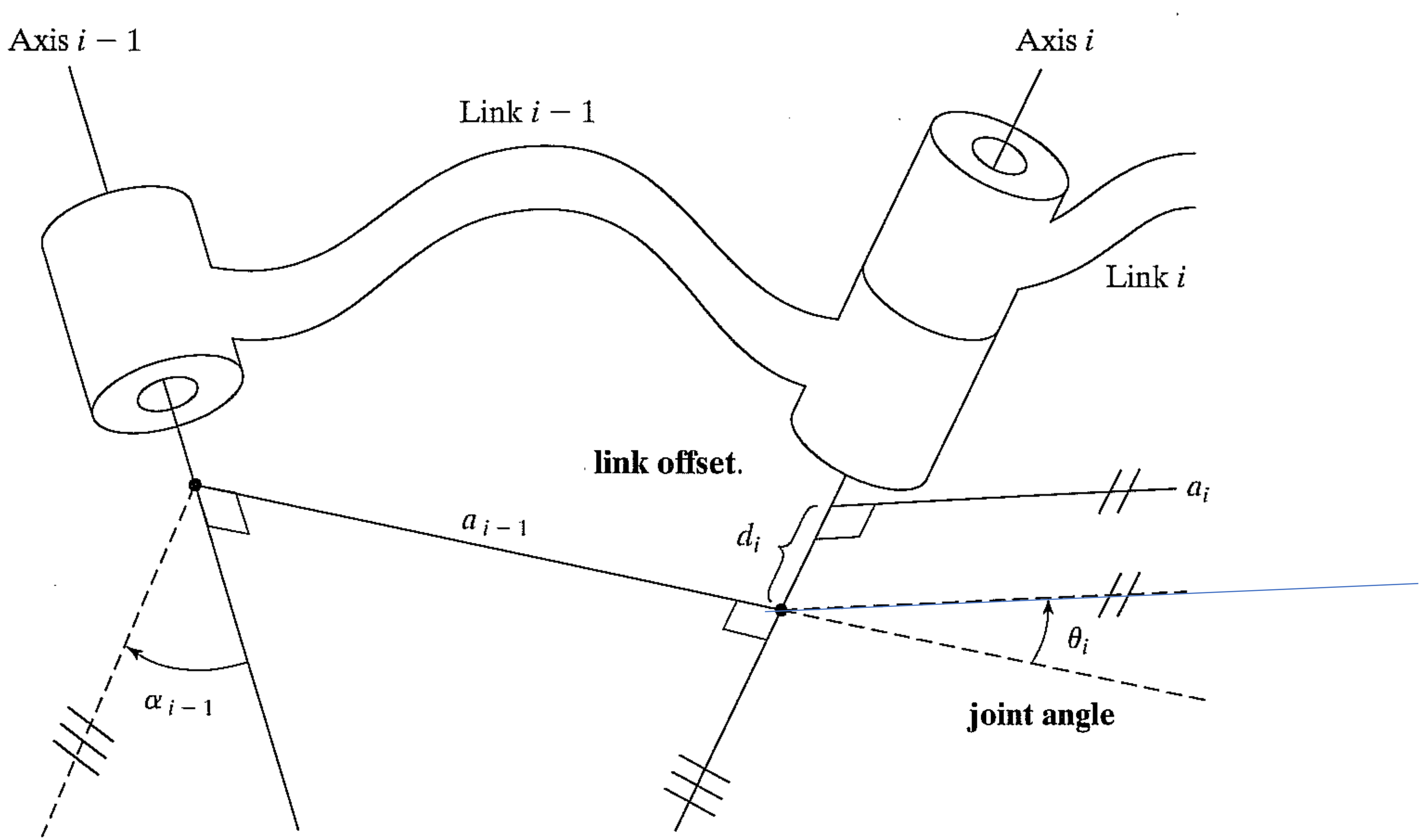
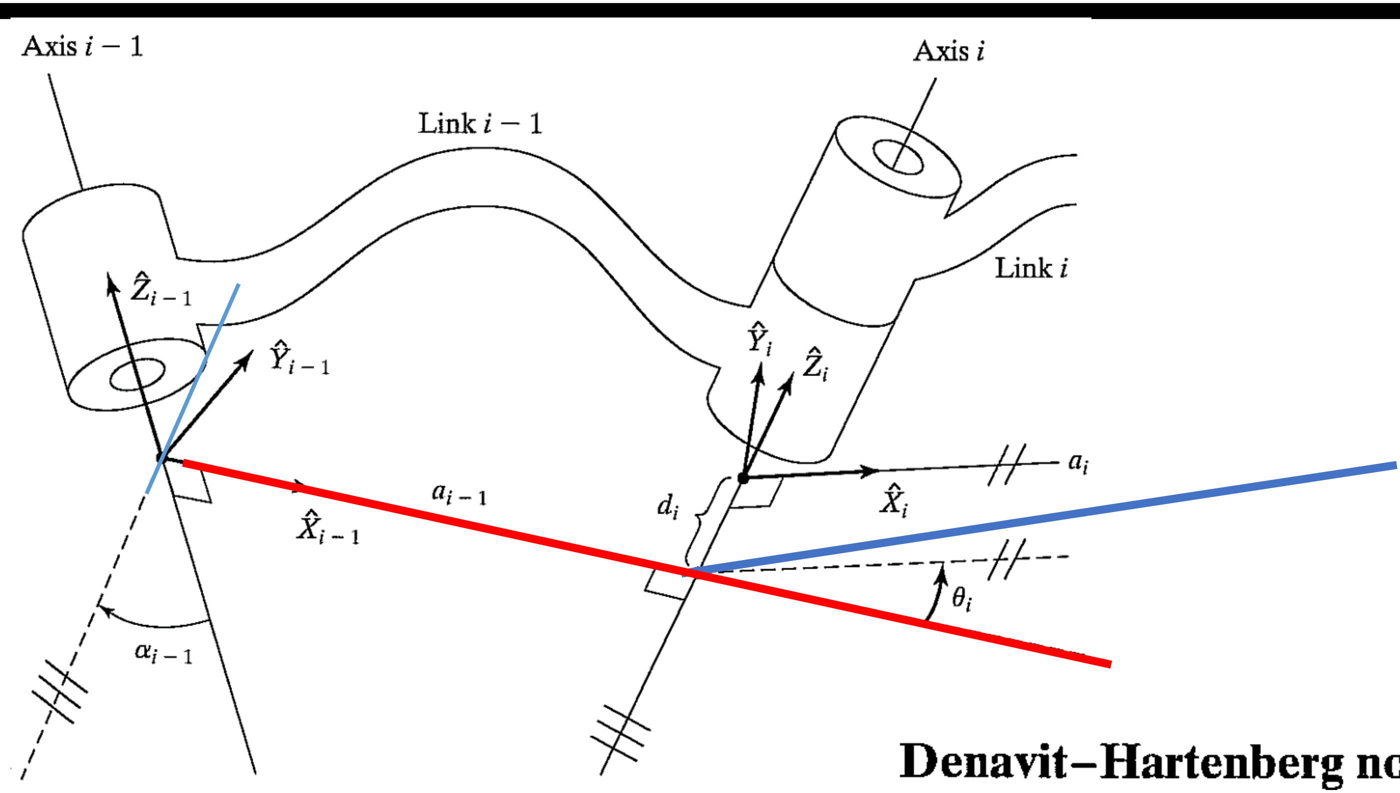


FIGURE 3.4: The link offset, d, and the joint angle, θ, are two parameters that may be used to describe the nature of the connection between neighboring links.



Denavit–Hartenberg notation

a_{i-1} = *the distance from* $\hat{Z}_{i-1}$ *to* $\hat{Z}_i$ *measured along* $\hat{X}_{i-1}$

α_{i-1} = *the angle from* $\hat{Z}_{i-1}$ *to* $\hat{Z}_i$ *measured about* $\hat{X}_{i-1}$

d_i = *the distance from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured along* $\hat{Z}_i$; and

θ_i = *the angle from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured about* $\hat{Z}_i$.

Intermediate links in the chain

The convention we will use to locate frames on the links is as follows: The $\hat{Z}$-axis of frame $\{i\}$, called $\hat{Z}_i$, is coincident with the joint axis i. The origin of frame $\{i\}$ is located where the a_i perpendicular intersects the joint i axis. $\hat{X}_i$ points along a_i in the direction from joint i to joint $i+1$.

In the case of $a_i = 0$, $\hat{X}_i$ is normal to the plane of $\hat{Z}_i$ and $\hat{Z}_{i+1}$. We define α_i as being measured in the right-hand sense about $\hat{X}_i$, and so we see that the freedom of choosing the sign of α_i in this case corresponds to two choices for the direction of $\hat{X}_i$. $\hat{Y}_i$ is formed by the right-hand rule to complete the ith frame. Figure 3.5 shows the location of frames $\{i-1\}$ and $\{i\}$ for a general manipulator.

3.4 CONVENTION FOR AFFIXING FRAMES TO LINKS

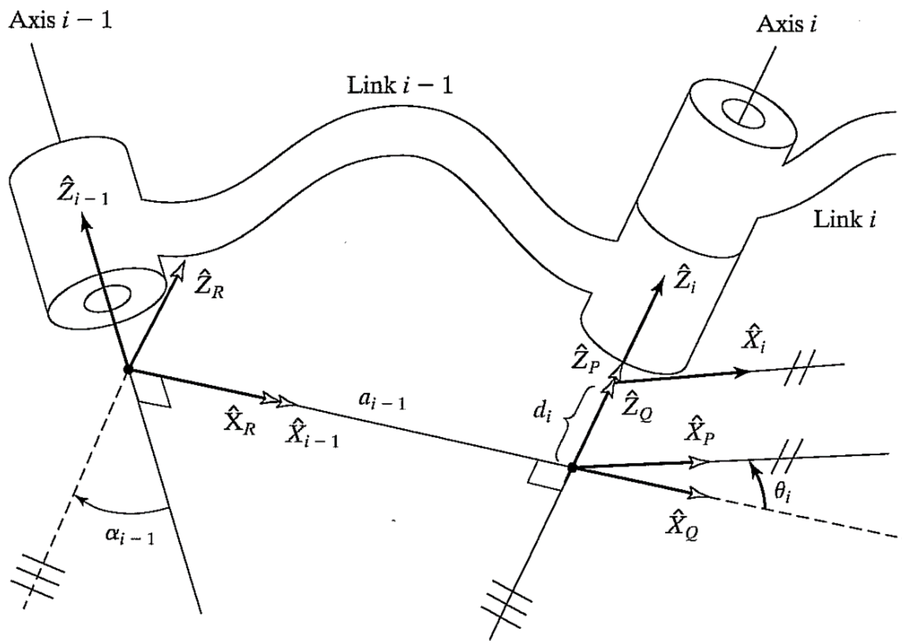
First and last links in the chain

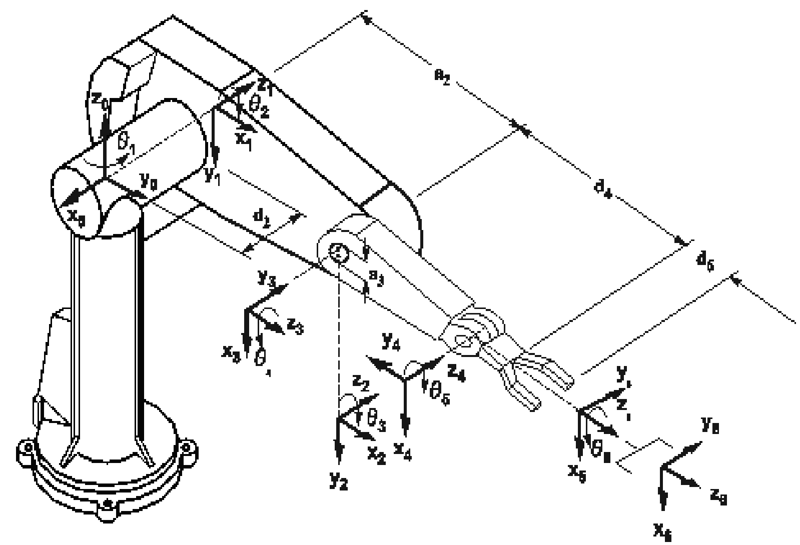
We attach a frame to the base of the robot, or link 0, called frame {0}. This frame does not move; for the problem of arm kinematics, it can be considered the reference frame. We may describe the position of all other link frames in terms of this frame.

Frame {0} is arbitrary, so it always simplifies matters to choose $\hat{Z}_0$ along axis 1 and to locate frame {0} so that it coincides with frame {1} when joint variable 1 is zero. Using this convention, we will always have $a_0 = 0.0$, $\alpha_0 = 0.0$. Additionally, this ensures that $d_1 = 0.0$ if joint 1 is revolute, or $\theta_1 = 0.0$ if joint 1 is prismatic.

For joint n revolute, the direction of $\hat{X}_N$ is chosen so that it aligns with $\hat{X}_{N-1}$ when $\theta_n = 0.0$, and the origin of frame $\{N\}$ is chosen so that $d_n = 0.0$. For joint n prismatic, the direction of $\hat{X}_N$ is chosen so that $\theta_n = 0.0$, and the origin of frame $\{N\}$ is chosen at the intersection of $\hat{X}_{N-1}$ and joint axis n when $d_n = 0.0$.

Derivation of link transformations





$$^{i-1}_{i}T = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T.$$

$$^{i-1}_{i}T = R_X(\alpha_{i-1}) D_X(a_{i-1}) R_Z(\theta_i) D_Z(d_i),$$

$$^{0}_{N}T = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T \ldots {}^{N-1}_{N}T.$$

$$^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

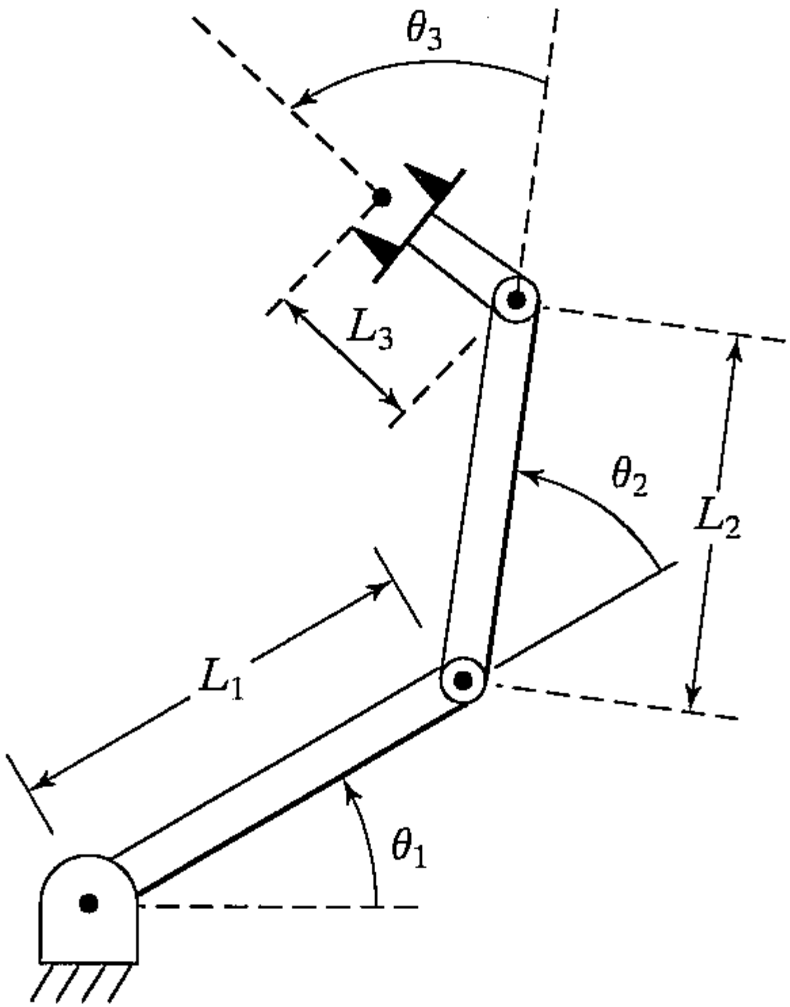
Summary of link-frame attachment procedure

The following is a summary of the procedure to follow when faced with a new mechanism, in order to properly attach the link frames:

1. Identify the joint axes and imagine (or draw) infinite lines along them. For steps 2 through 5 below, consider two of these neighboring lines (at axes i and $i+1$).
2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ith axis, assign the link-frame origin.
3. Assign the $\hat{Z}_i$ axis pointing along the ith joint axis.
4. Assign the $\hat{X}_i$ axis pointing along the common perpendicular, or, if the axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes.
5. Assign the $\hat{Y}_i$ axis to complete a right-hand coordinate system.
6. Assign {0} to match {1} when the first joint variable is zero. For $\{N\}$, choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to become zero.

RRR (or 3*R*) mechanism.

general configration all joints ≠ zero



1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^0_3T$?

In page

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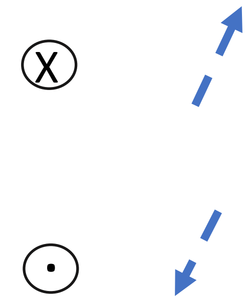


zero conf.>>>all joint =zero
if you assigned frams correctly>>all Xs in same direction.

See notes

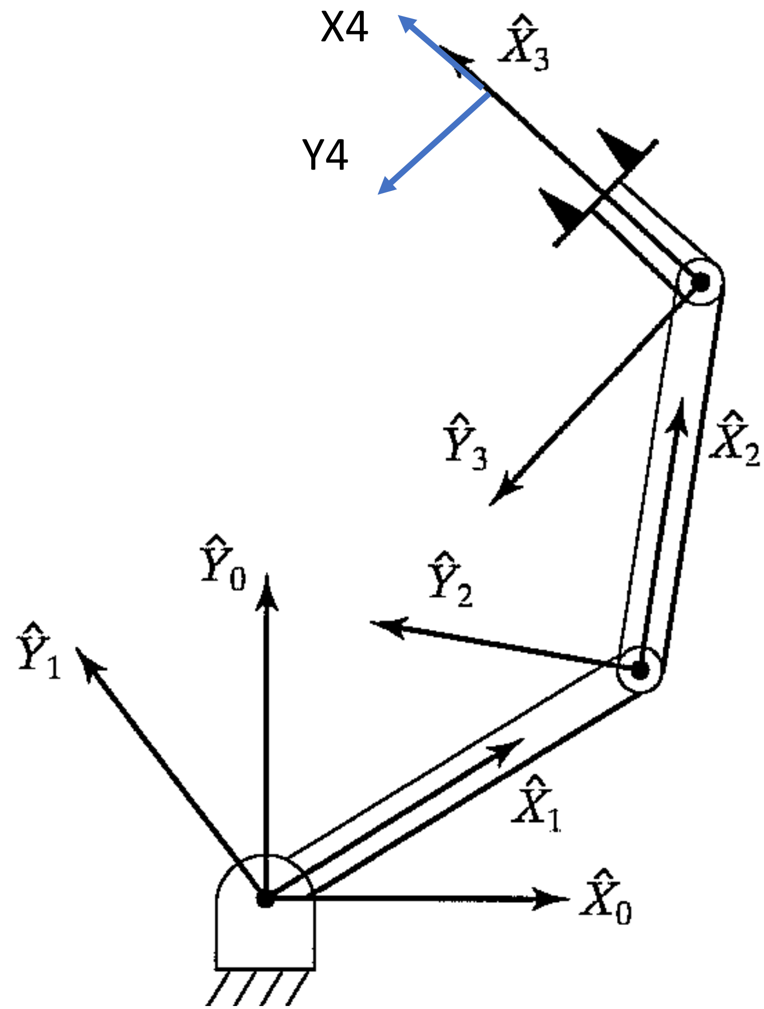
RRR (or 3*R*) mechanism.

general configration all joints ≠ zero

1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^{0}_{3}T$?



See notes

RRR (or 3*R*) mechanism.

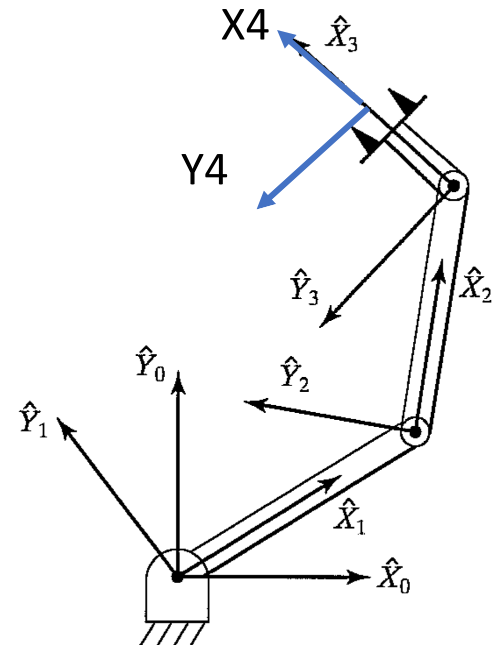
general configration all joints ≠ zero

1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^{0}_{3}T$?

i	α_{i-1}	a_{i-1}	d_i	θ_i	i-1
1	0	0	0	θ_1	0
2	0	L_1	0	θ_2	1
3	0	L_2	0	θ_3	2



RRR (or 3*R*) mechanism.

general configration all joints ≠ zero

1-Assign frames on robot based on DH convention

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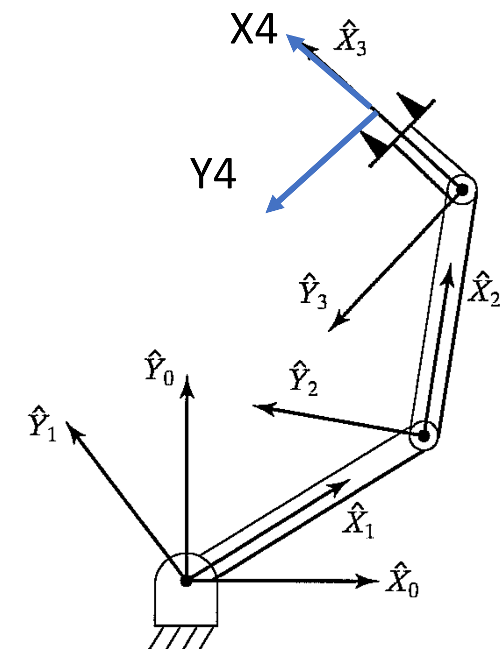
3-find ${}^{0}_{3}T$?



C12=cos(θ1+ θ2)≠ C1C2 ≠C1+C2

$$
{}^{0}_{3}T = \begin{bmatrix} C_{123} & -S_{123} & 0 & l_1C_1 + l_2C_{12} \\ S_{123} & C_{123} & 0 & l_1S_1 + l_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

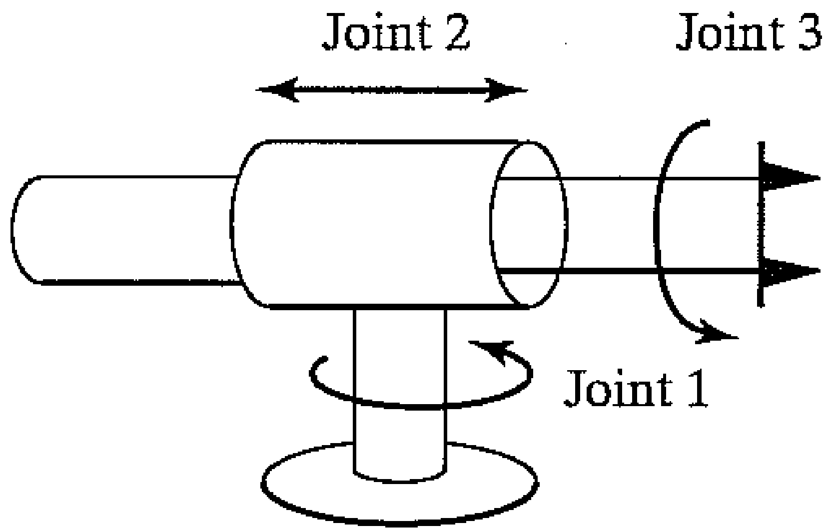
RPR mechanism

general configration all joints ≠ zero

1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^{0}_{3}T$?

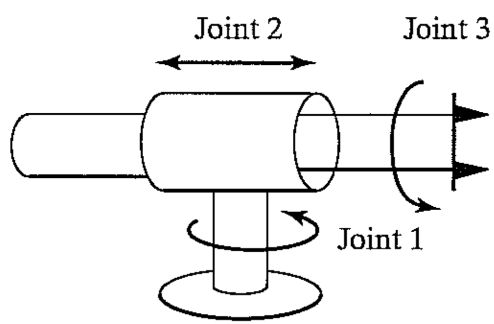


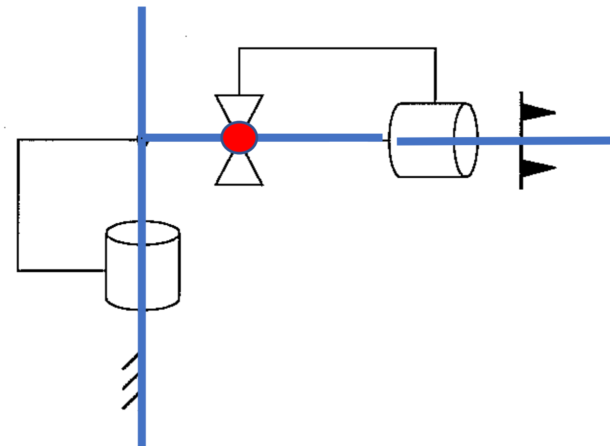
See notes

zero conf.>>>all joint =zero
if you assigned frams correctly>>all Xs in same direction.

RPR mechanism

general configration all joints ≠ zero

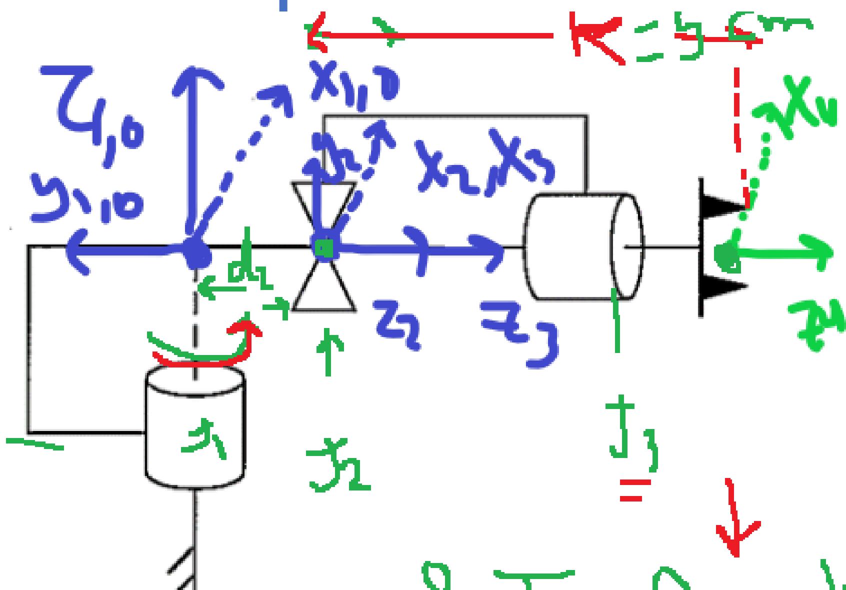




1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find $_3^0T$?



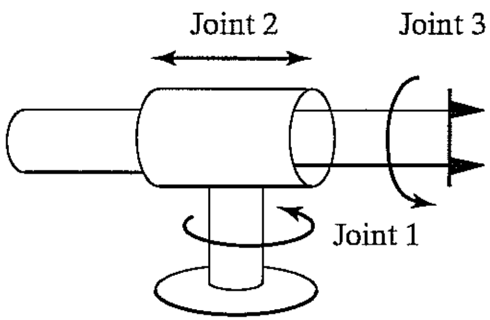
ee notes

$$_3^0T = {}_1^0T \; {}_2^1T \; {}_3^2T$$

zero conf.>>>all joint =zero
if you assigned frams correctly>>all Xs in same direction.

RPR mechanism

general configration all joints ≠ zero



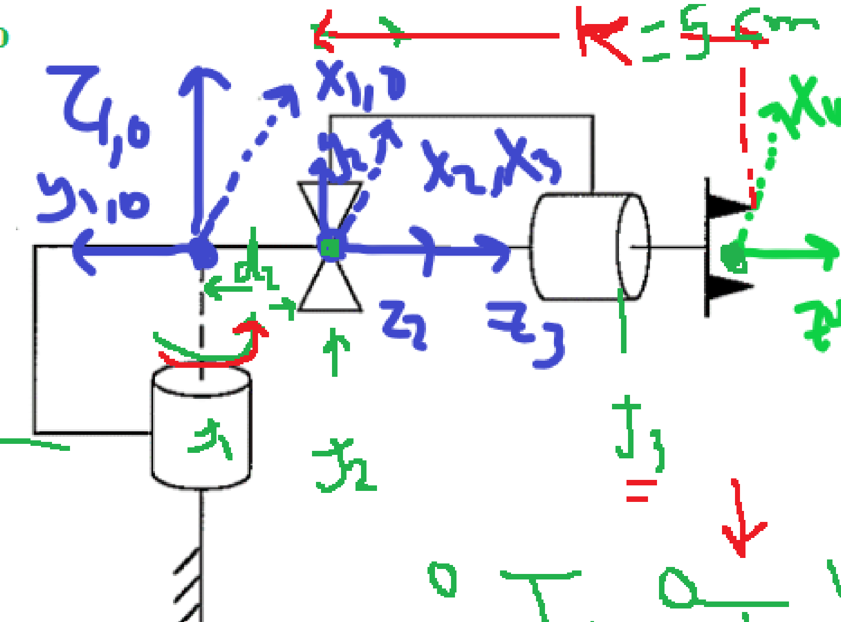
1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^0_3T$?

general configration all joints ≠ zero

i+1	α	a	d	θ	i
1	0	0	0	θ1(t)	0
2	90	0	d2(t)	0	1
3	0	0	0	θ3(t)	2
4	0	0	K	Zero	3



$${}^0_3T = {}^0_1T \; {}^1_2T \; {}^2_3T$$

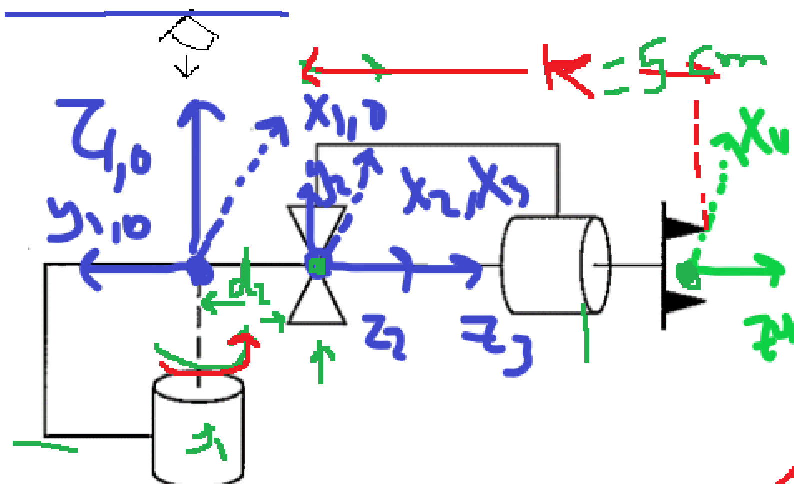
zero conf.>>>all joint =zero
if you assigned frams correctly>>all Xs in same direction.

RPR mechanism

1-Assign frames on robot based on DH convention

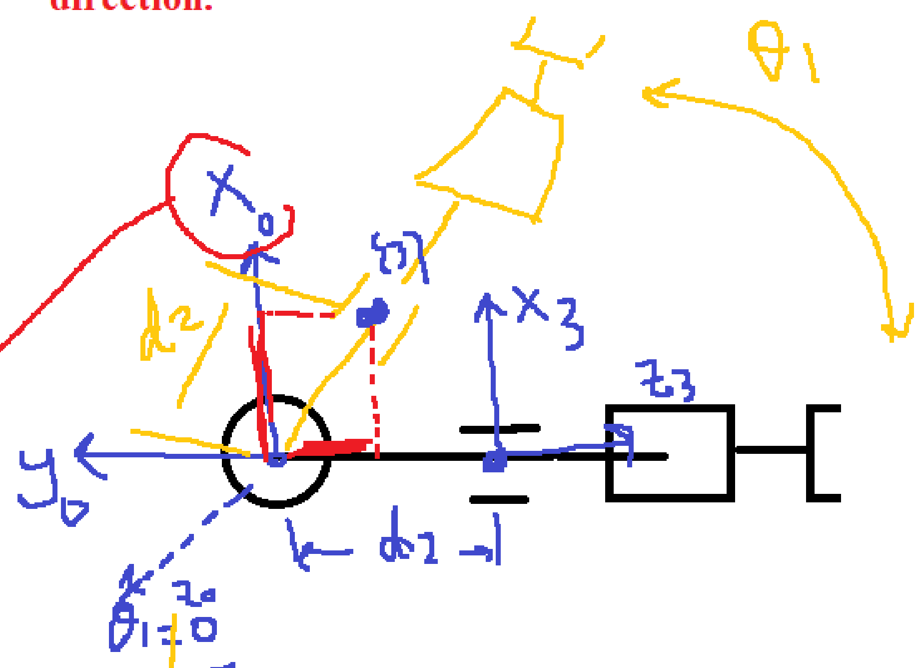
2-fill in the DH parameters table

general configration all joints ≠ zero



$$ {}^{0}_{3}P = \begin{bmatrix} d_2 S_1 \\ -d_2 C_1 \\ 0 \end{bmatrix} $$

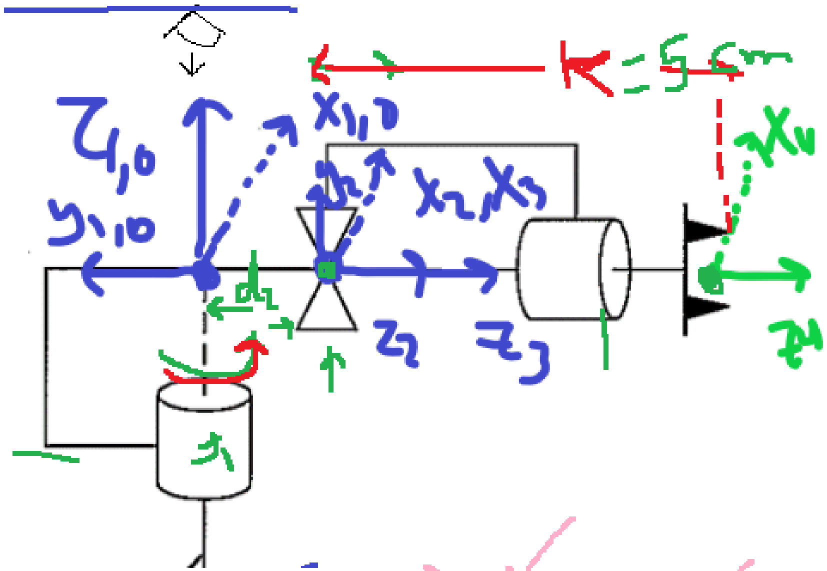
zero conf.>>>all joint =zero
if you assigned frams correctly>>all Xs in same direction.



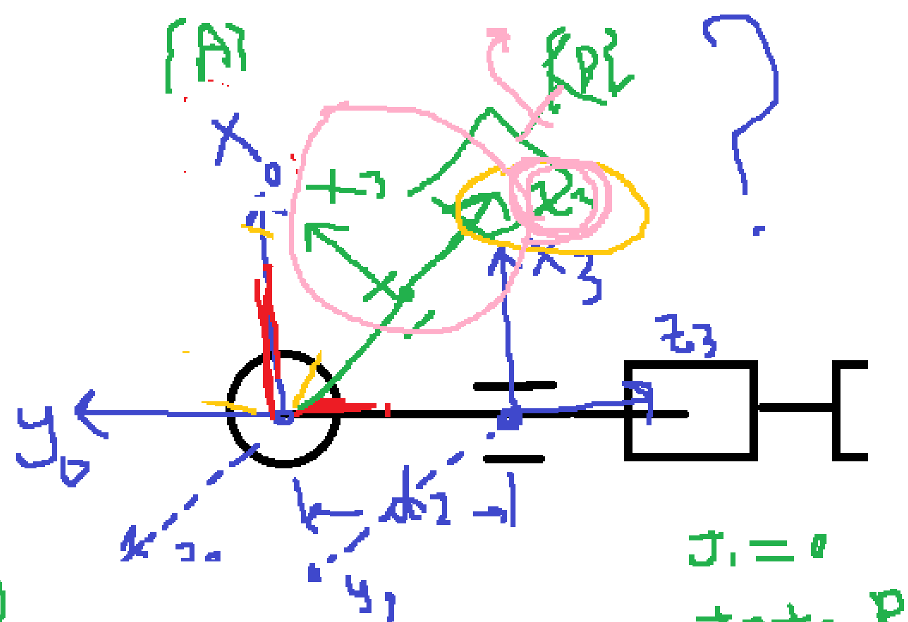
RPR mechanism

3-find ${}^0_3T$?

general configration all joints ≠ zero



zero conf>>>all joint =zero
if you assigned frams correctly>>all Xs in same direction.



$${}^0_3R = R_x(90)\, R_y(\theta_1)\, R_z(\theta_3) = R_{xyz}(90, \theta_1, \theta_3)$$

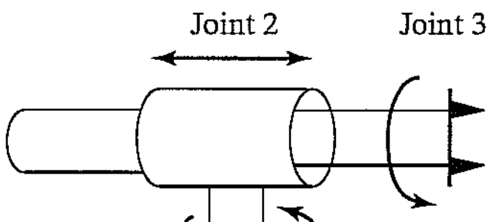
Euler

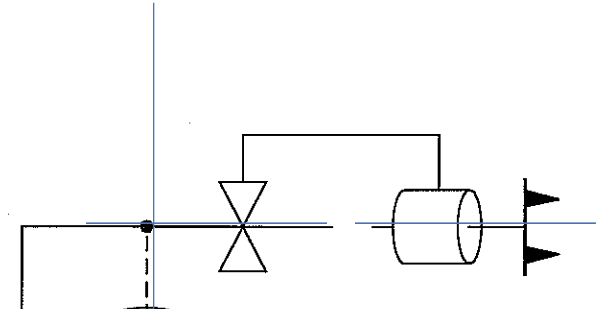
$J_1 = 0$

$J_2 \neq 0$ P

$J_3 = 0$

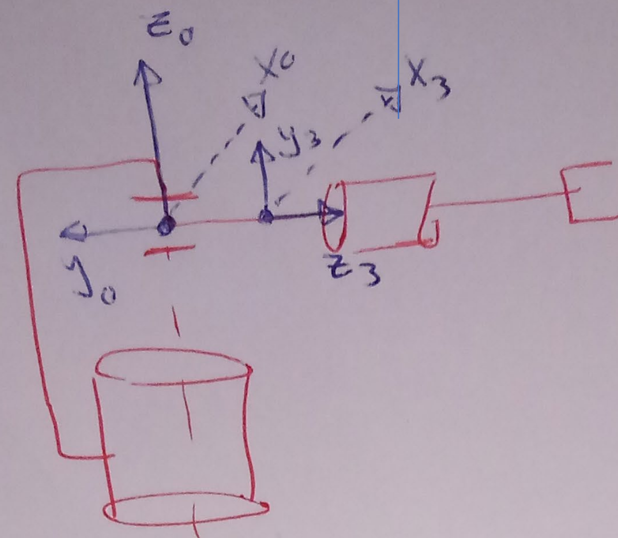
RPR mechanism





1-Assign frames on robot based on DH convention

2-fill in the DH parameters table



$$
{}^{0}_{3}T = \begin{bmatrix} C_1C_3 & -C_1S_3 & S_1 & S_1 d_2 \\ S_1C_3 & -S_1S_3 & -C_1 & -C_1 d_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

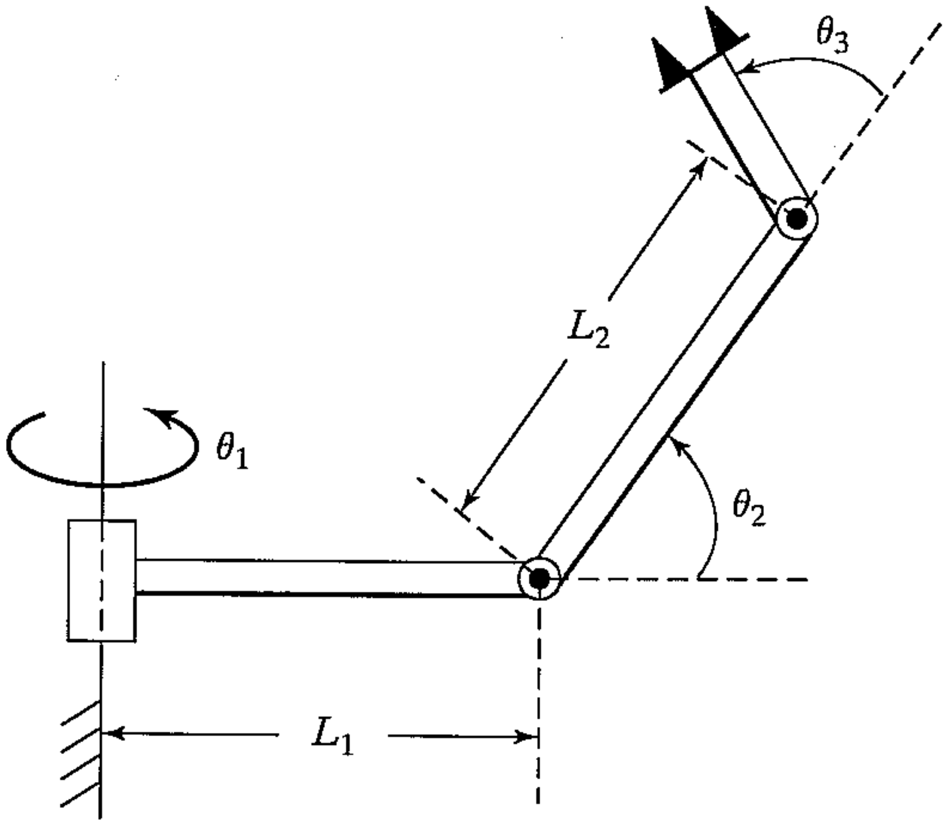


FIGURE 3.29: The $3R$ nonplanar arm (Exercise 3.3).

1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^{0}_{3}T$?

See notes

1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

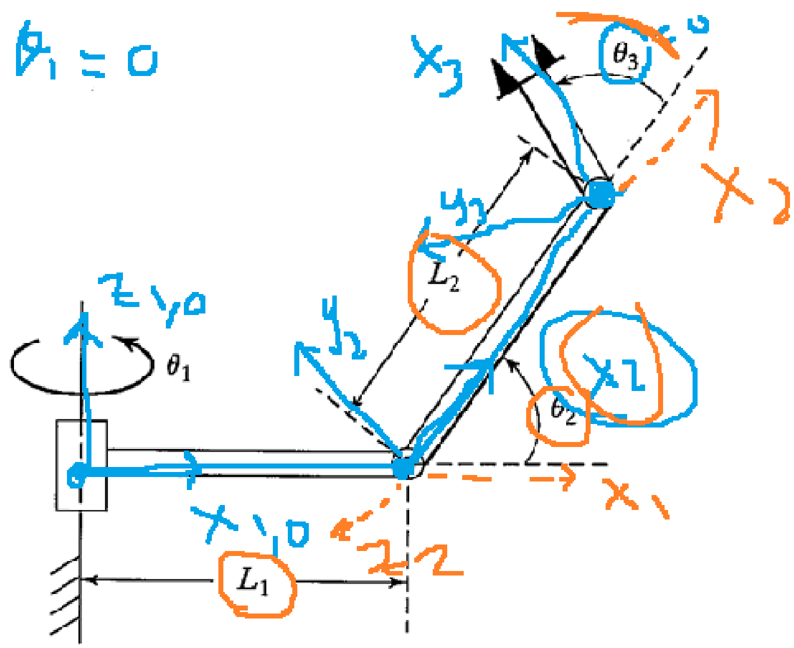
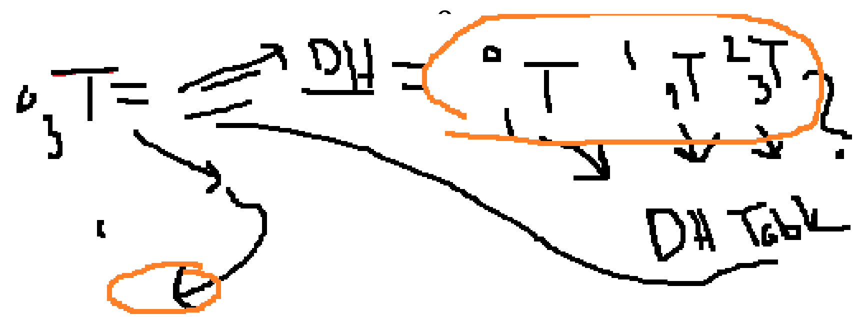


FIGURE 3.29: The 3R nonplanar arm (Exercise 3.3).

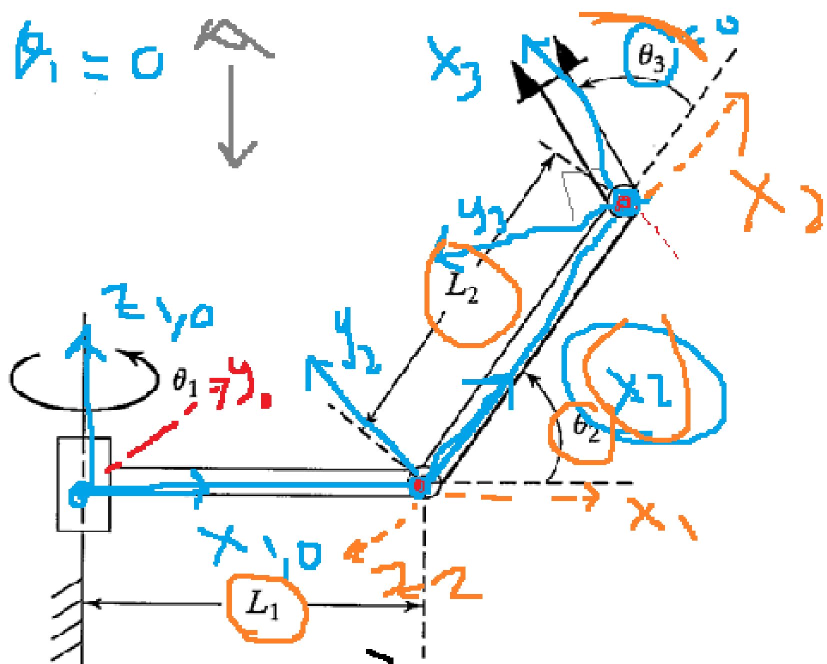


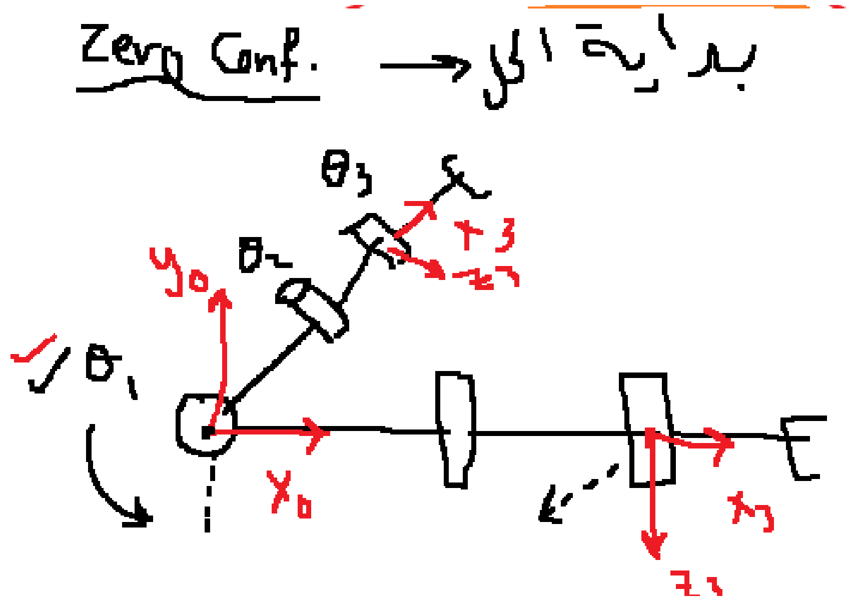
i+1	α	a	d	θ	i
1	0	0	0	$\theta_1(t)$	0
2	90	L_1	0	$\theta_2(t)$	1
3	0	L_2	0	$\theta_3(t)$	2

1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^0_3T$?





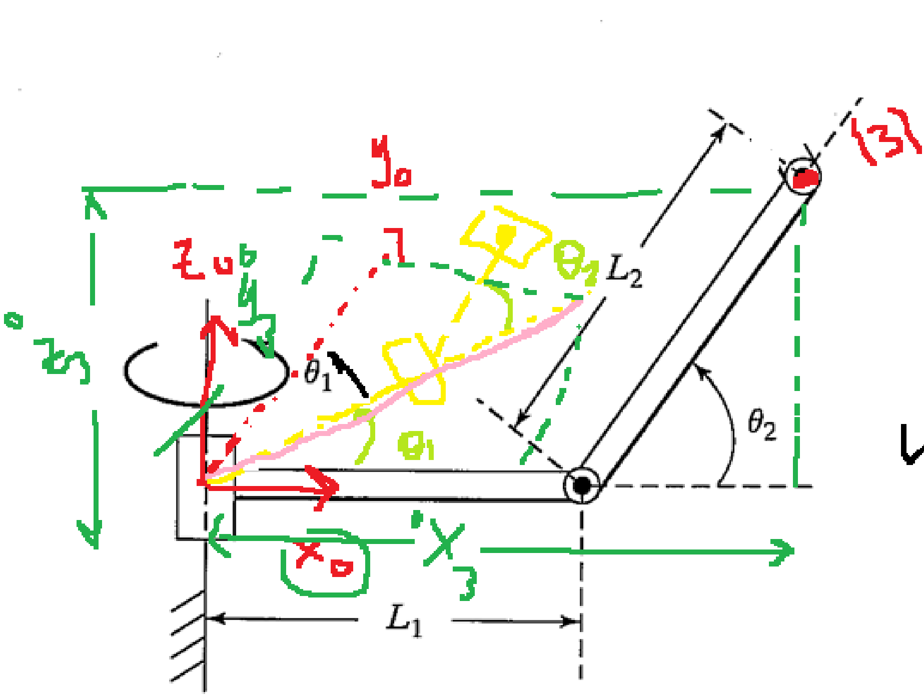
$$ {}^0_3T = \begin{bmatrix} {}^0_3R & {}^0_3P \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} $$

$$ {}^0_3R = R_{xyz\ Eular} = R_x(90)\, R_y(\theta_1)\, R_z(\theta_2+\theta_3) $$

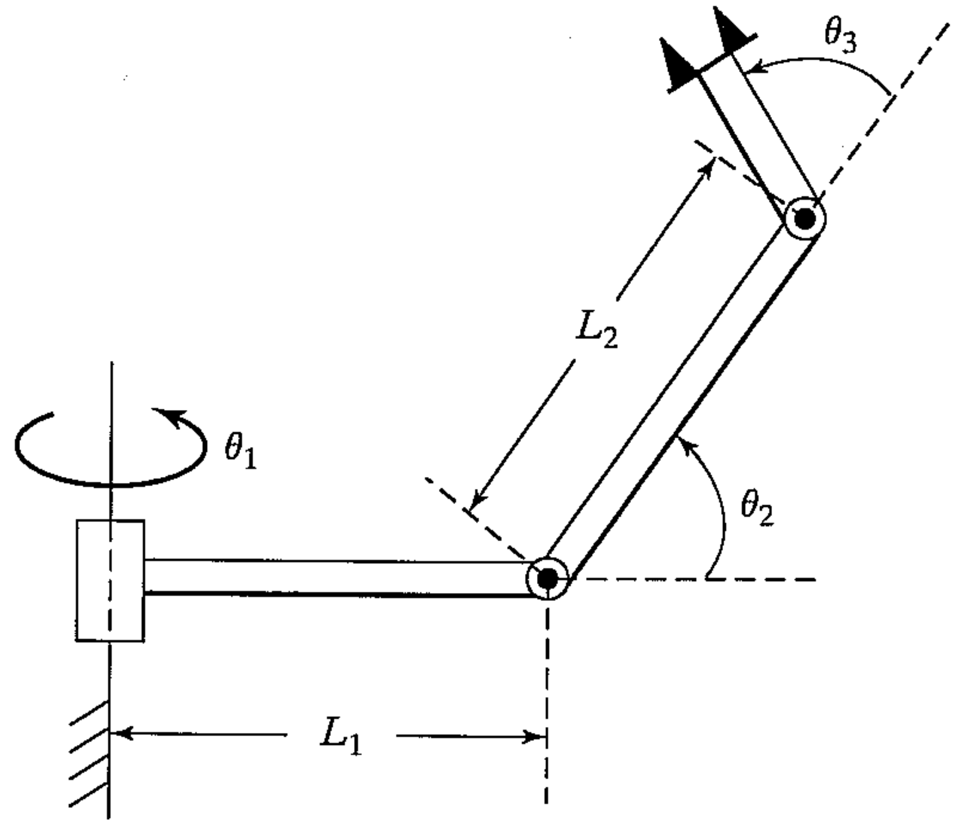
1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^{0}_{3}T$?



$${}^{0}_{3}P = \begin{bmatrix} {}^{0}_{3}x = (L_1 + L_2C_2)C_1 \\ {}^{0}_{3}y = (L_1 + L_2C_2)S_1 \\ {}^{0}_{3}z = L_2 S_2 \end{bmatrix}$$



1-Assign frames on robot based on DH convention

2-fill in the DH parameters table

3-find ${}^0_3T$?

FIGURE

$$
{}^0_3T = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & (L_1+L_2C_2)C_1 \\ S_1S_{23} & -S_1S_{23} & +C_1 & (L_1+L_2C_2)S_1 \\ S_{23} & C_{23} & 0 & L_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

See note

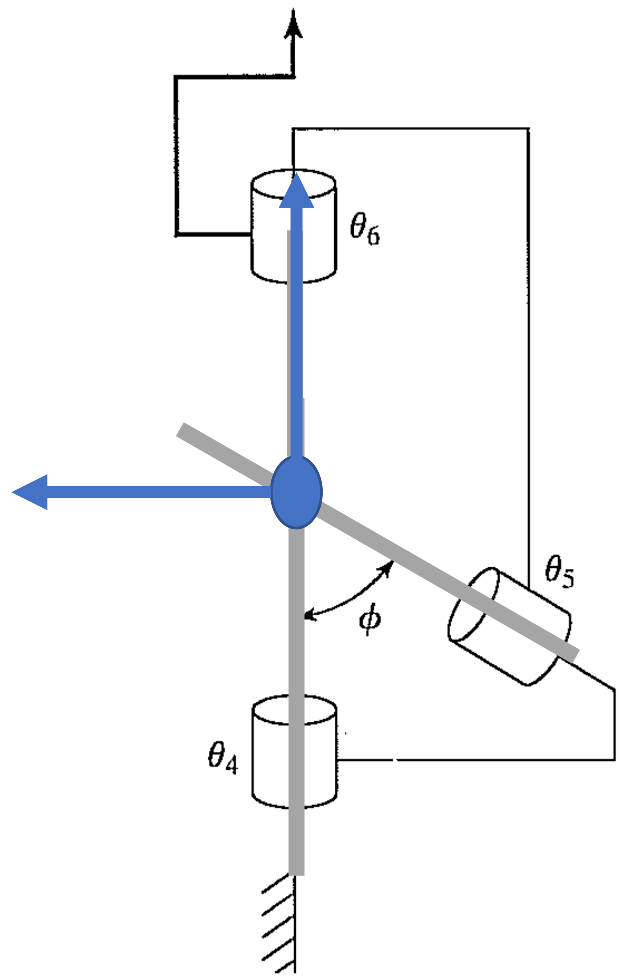
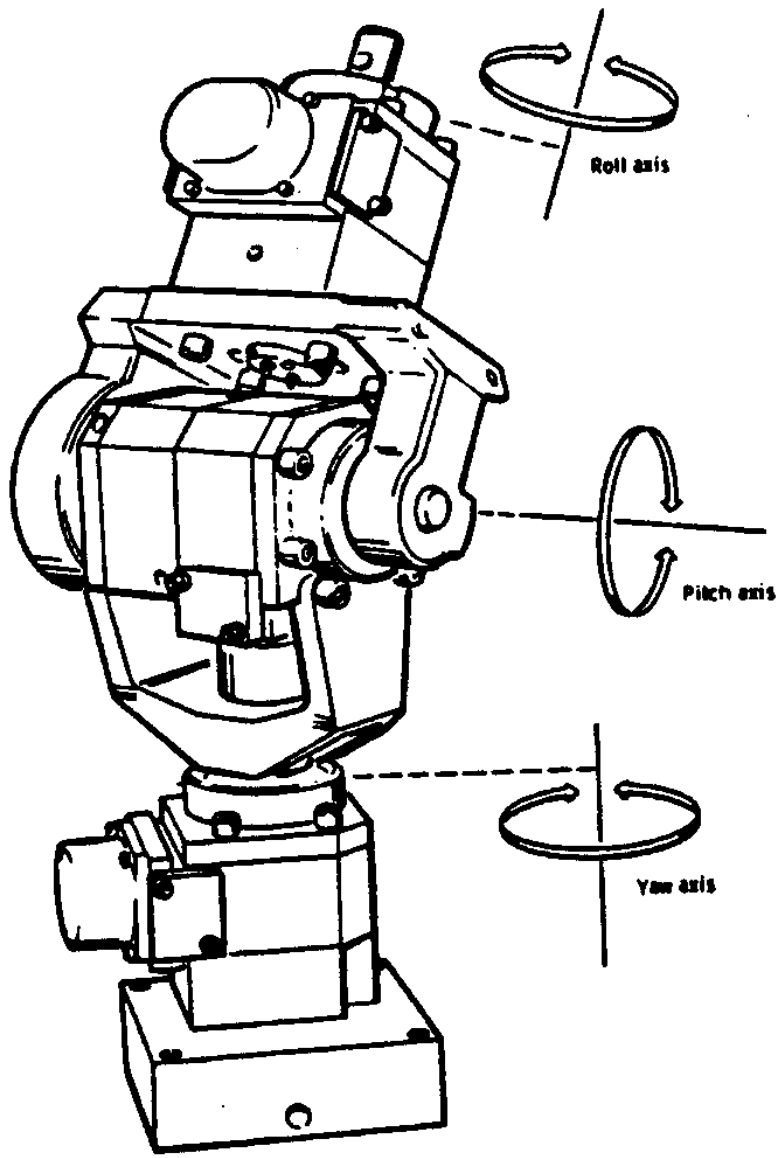


FIGURE 3.33: $3R$ nonorthogonal-axis robot (Exercise 3.11).

See notes


Roll axis
Pitch axis
Yaw axis

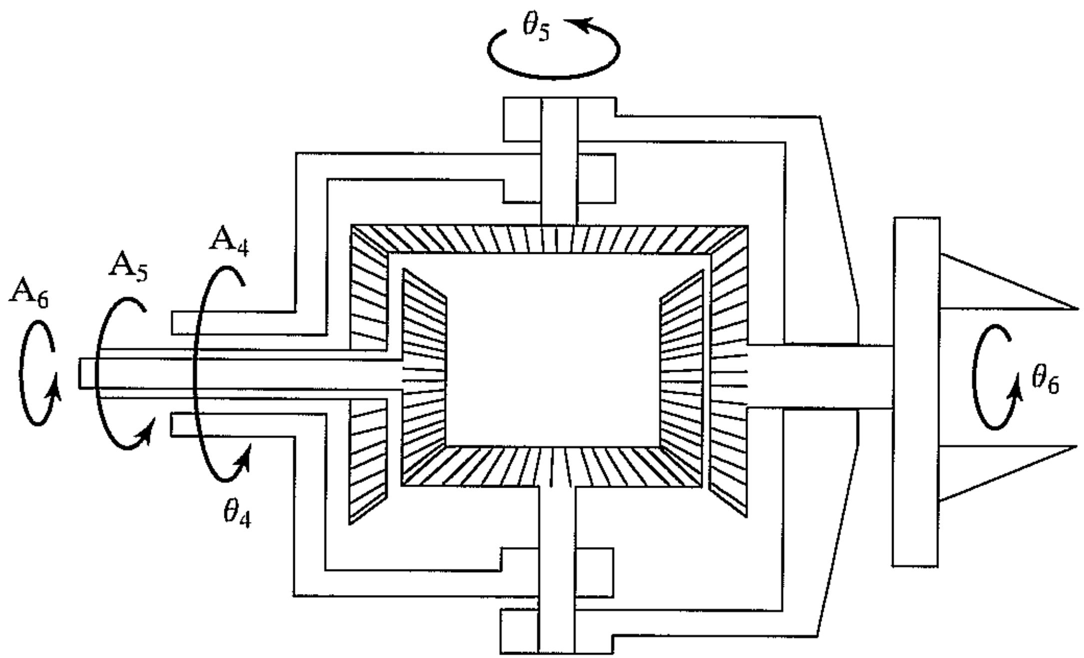
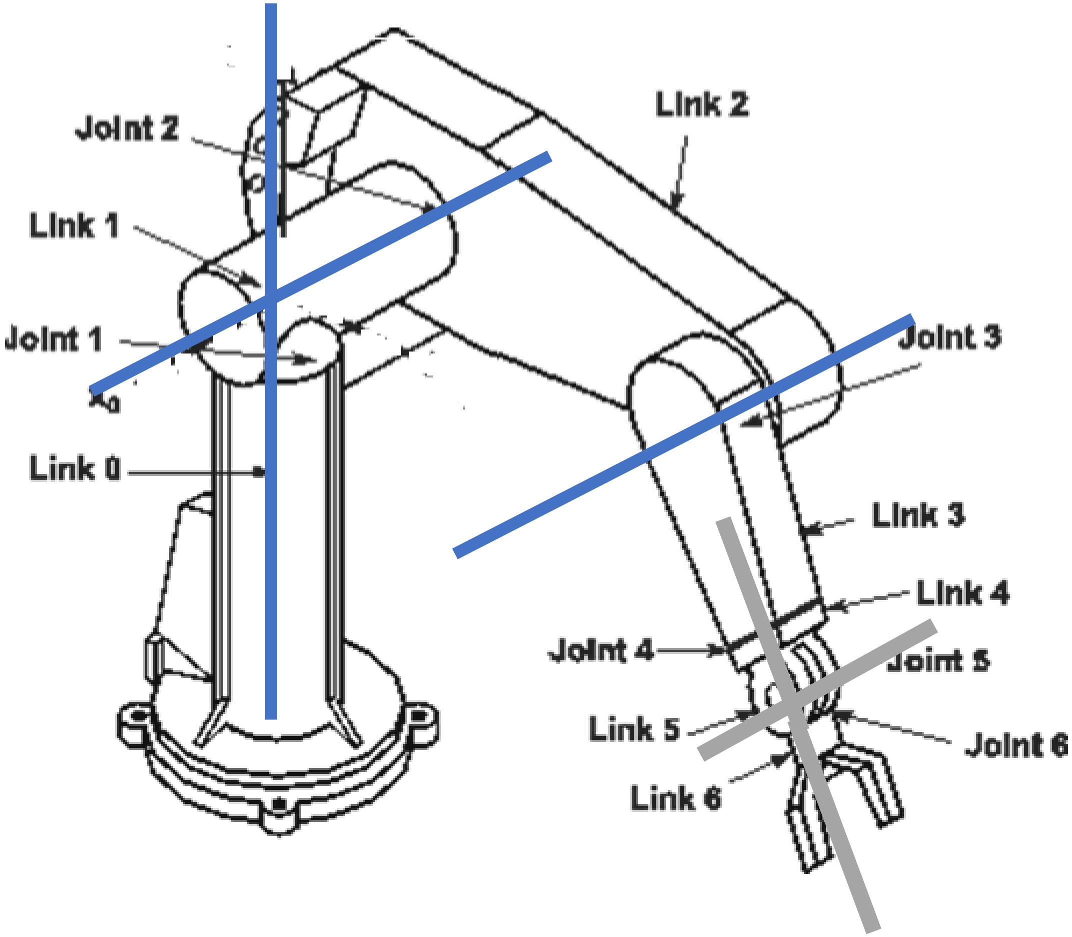
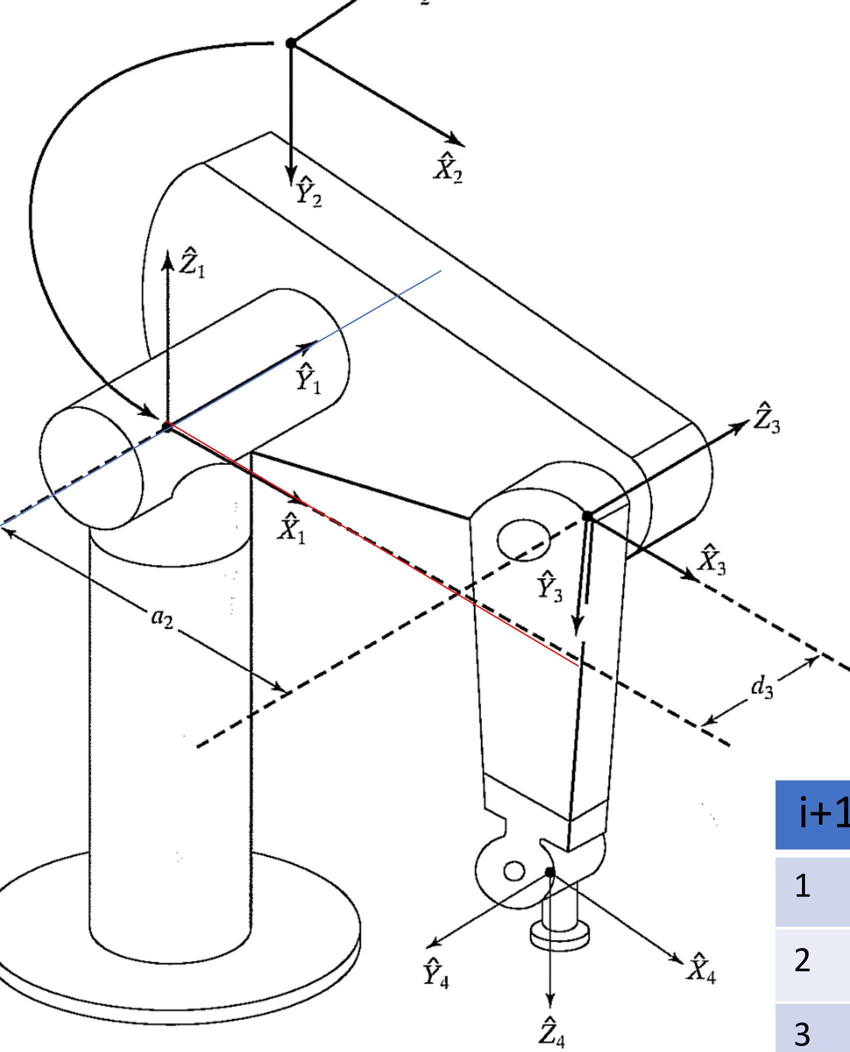
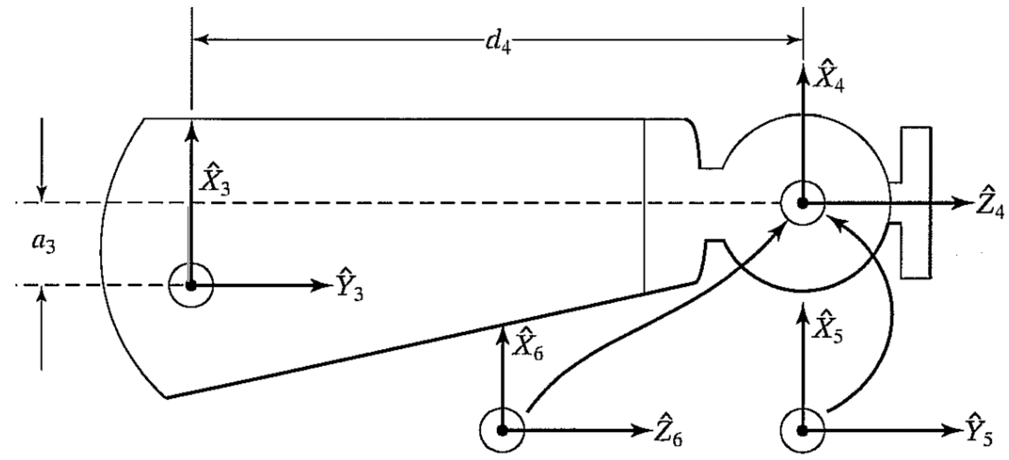


FIGURE 8.8: An orthogonal-axis wrist driven by remotely located actuators via three concentric shafts.



PUMA robot 6DOF





i+1	αi-1	ai-1	di	Theta	i
1	0	0	0	θ_1	0
2	-90	0	0	θ_2	1
3	0	a2	d3	θ_3	2
4	-90	a3	d4	θ_4	3
5	90	0	0	θ_5	4
6	-90	0	0	θ_6	5

See notes

$$r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6),$$
$$r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6),$$
$$r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6,$$

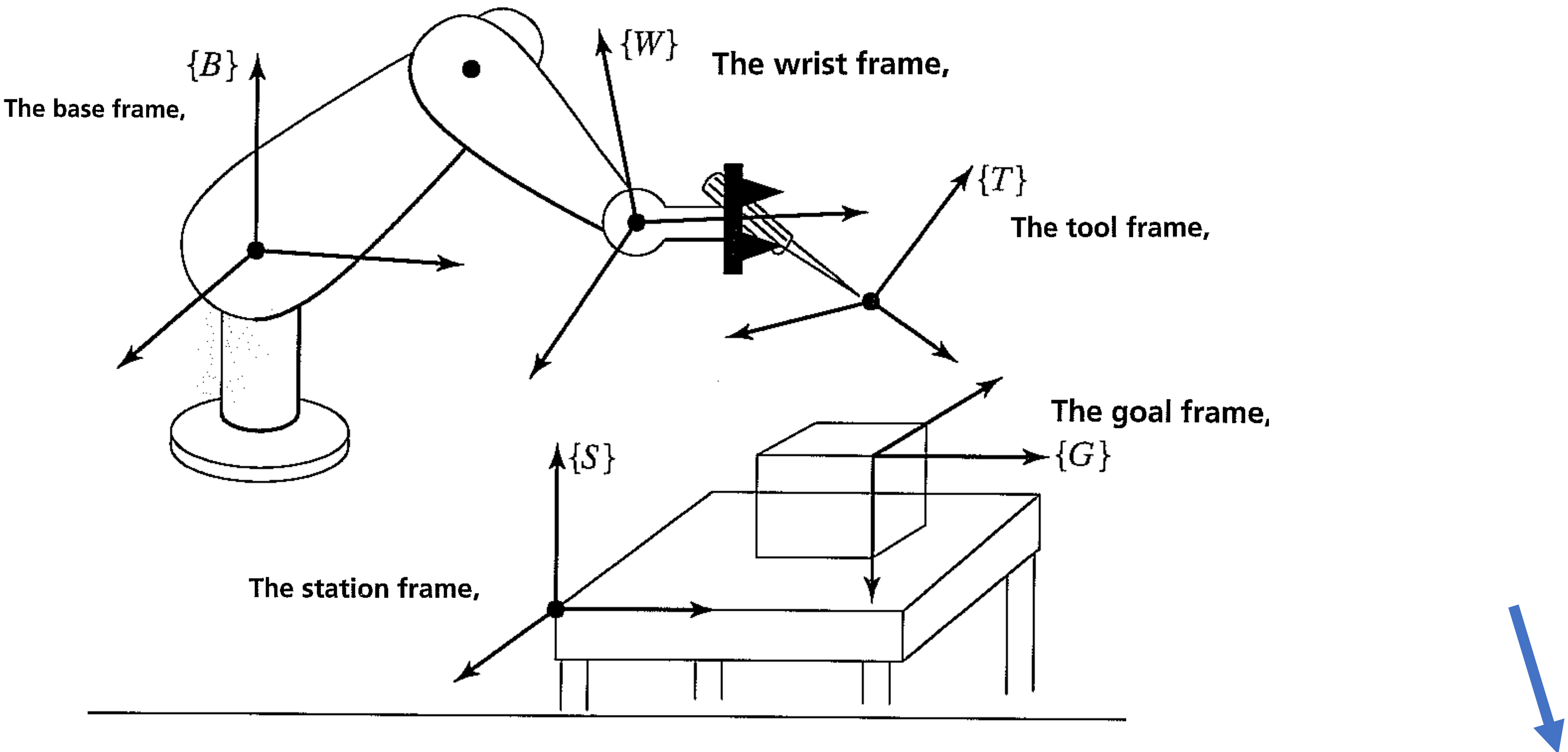
$$r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6),$$
$$r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6),$$
$$r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6,$$

$$r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5,$$
$$r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5,$$
$$r_{33} = s_{23}c_4s_5 - c_{23}c_5,$$

$$p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1,$$
$$p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1,$$
$$p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}.$$

$${}^0_6T = {}^0_1T\,{}^1_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

3.8 FRAMES WITH STANDARD NAMES



3.6 ACTUATOR SPACE, JOINT SPACE, AND CARTESIAN SPACE

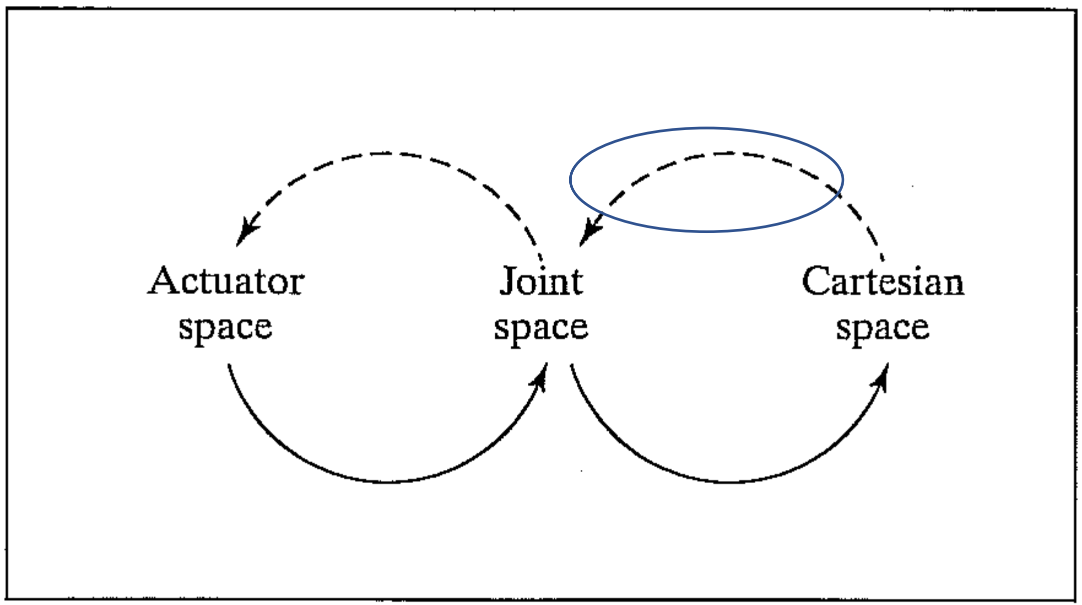


FIGURE 3.16: Mappings between kinematic description